UNIT - II STATISTICAL ASPECTS OF FATIGUE BEHAVIOUR

LOW CYCLE FATIGUE (LCF) & HIGH CYCLE FATIGUE (HCF)

LCF → 0 above to 10^3 or 10^4 cycles

HCF → 10^3 or 10^4 cycles to 10^8 cycles.

Fatigue problems can be conveniently divided into two categories namely,

→ Those which involve high strain (or deformation) cycling

→ Those which involve load (or nominal stress) cycling

* Fatigue curves may be obtained by testing plain specimens in laboratory under conditions of either constant strain cycling or by the more conventional constant load cycling.

* When the stress levels do not significantly exceed the yield strength of the material, the two curves are more or less identical.

* On the other hand where high strain & plasticity is involved stress-strain relationship is no longer linear & is further complicated under cyclic loading conditions by the cyclic strain hardening or strain softening characteristic executed by the material.

* It therefore becomes necessary to distinguish between high strain LCF and low stress HCF.

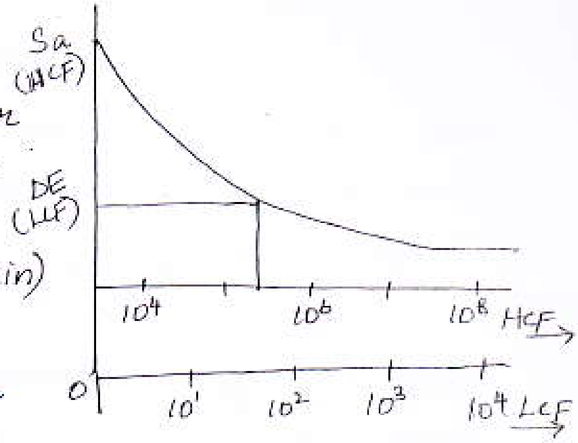
* LCF is obt'd by testing specimens under conditions of constant strain (or) deformation

* HCF is obt'd by testing specimens under conditions of constant load.

HCF:

In HCF a large number of cycles is required to cause failure. HCF refer to those combination of stress (or strain) and number of cycles during which macroscopic plasticity (or) yielding does not occurs. Under these conditions, the usual stress-strain relationship applies and hence HCF curves are usually represented by stress amplitude (S_a) Vs number of cycles to failure (N_f) plot.

LCF:



In LCF only a small number of cycles is required to cause failure. It refers to those combinations of stress (or strain) and no. of cycles during which considerable macroscopic plasticity occurs. Under these circumstances linear stress-strain relationship is invalid and hence LCF curves are represented by strain range ΔE Vs no. of cycles to failure (N_f) plot.

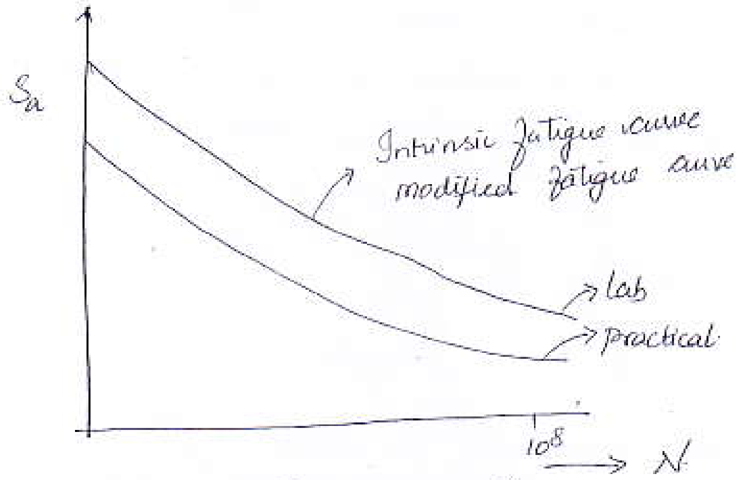
FACTORS INFLUENCING FATIGUE BEHAVIOUR:

In any fatigue analysis of a component whether this be for LCF or HCF region, many influencing factors which affect the fatigue behaviour are

1) Type and nature of loading.
2) Size of component and stress-strain distribution
3) Surface finish & directional (components) properties
4) Stress-strain concentrations.
5) Mean stress or strain
6) Environmental effects.
7) Metallurgical factors & properties
8) Strain rate & frequency effects.

DESIGNING AGAINST HCF:

→ It is useful to consider the presently used Engineering approach to design against fatigue in the intermediate and high cycle region.



→ The upper curve in the fig, represents the intrinsic fatigue - curve for the material (i.e) the curve which is obtained in lab from standard tests on apparently identical specimen showing a polished surface & of a standard size.

→ The test may be performed at a particular temperature and/or any other environmental conditions of interest.

→ The tests may be such that mostly the conditions of completely reversed constant amplitude loading is used.

→ Before this intrinsic fatigue curve may be applied to a real component, it must be modified to account for those factors not included in the laboratory test but present in the component, for a component subjected to a 1D constant amplitude.

→ For a component subjected to a 1D constant

→ Completely reversed cyclic loading conditions S_a' is given by $S_a' = \frac{S_a}{K_f} K_s C_s C_L$ at a particular life under consideration.

where S_a' → modified fatigue strength of a component corresponding to a particular life cycle.

S_a → Intrinsic fatigue strength

K_s → Surface finish factor

C_s → Size factor

C_L → Loading factor

K_f → fatigue strength reduction factor.

<u>Strain-based approach to total life</u>

→ Coffin & Manson in 1954 worked independently on thermal fatigue problems & they proposed a characterization of fatigue life based on the plastic strain amplitude.

→ They noted that when the log of the plastic strain amplitude $\left(\frac{\Delta\epsilon_p}{2}\right)$ was plotted against the logarithm of the number of load reversals to failure $2N_f$, a linear relationship resulted for metallic materials i.e

$$① \longleftarrow \frac{\Delta\epsilon_p}{2} = \epsilon_f'(2N_f)^c \longrightarrow \text{Coffin \& Manson relationship.}$$

where ϵ_f' is the fatigue ductility coefficient.

c is the fatigue ductility exponent.

→ In general ϵ_f' is approximately equal to the true fracture ductility ϵ_f in monotonic tension and c is in the range of -0.5 to -0.7 for most metals.

Eqn ① provides a convenient engineering expression for characterizing the total fatigue life.

Seperation of low cycle and high cycle fatigue lines

→ The total strain amplitude in a constant strain amplitude test $\Delta\epsilon/2$ can be written as the sum of elastic strain amplitude, $\Delta\epsilon_e/2$ and the plastic strain amplitude $\Delta\epsilon_p/2$

$$\frac{\Delta\epsilon}{2} = \frac{\Delta\epsilon_p}{2} + \frac{\Delta\epsilon_e}{2} \longrightarrow (2)$$

Using Basquin equation ie

$$\sigma_a = \frac{\Delta\sigma}{2} = \sigma_f'(2N_f)^b \xRightarrow{\text{We get}} \frac{\Delta\epsilon_e}{2} = \frac{\sigma_f'}{E}(2N_f)^b$$

The elastic ~~str~~ term can be written as

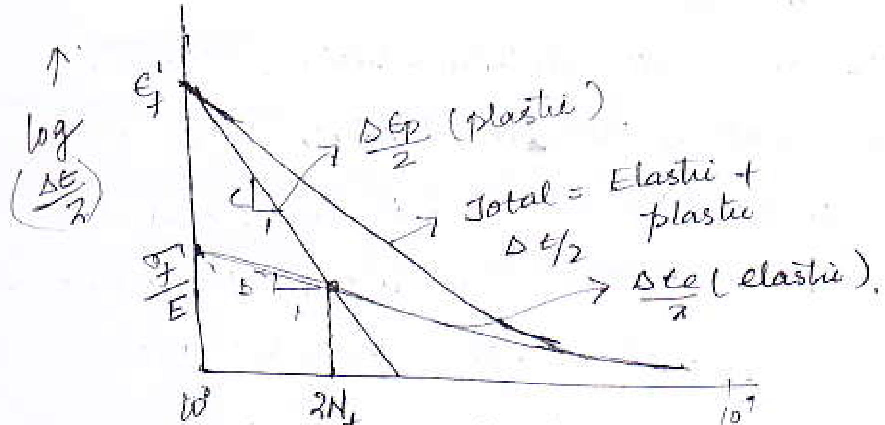
$$\frac{\Delta\epsilon_e}{2} = \frac{\Delta\sigma}{2E}$$

from eqn (1) $\quad \frac{\Delta\epsilon_p}{2} = \epsilon_f'(2N_f)^c \rightarrow$ plastic term.

The total strain can now be rewritten as

$$(2) \Rightarrow \frac{\Delta\epsilon}{2} = \underbrace{\frac{\sigma_f'}{E}(2N_f)^b}_{\text{elastic}} + \underbrace{\epsilon_f'(2N_f)^c}_{\text{plastic}} \longrightarrow (3).$$

Equation (3) is the basis of the strain-life method and is termed the strain-life relation.



(Strain - life curve.)

→ Eqn ③ is explained graphically. Both elastic & plastic relations are straight lines on a log-log plot.

→ The total strain amplitude $\Delta\epsilon/2$, can be plotted by simply summing the elastic & plastic values as shown in graph

→ At large strain amplitudes the strain-life curve approaches the plastic line & at low amplitudes, the curve approaches the elastic line

TRANSITION LIFE:

→ The variations of the elastic, plastic and total strain amplitudes are plotted in the above fig., as functions of the number of load reversals to failure $2N_f$.

→ In order to examine the implications of the above plot, for short and long fatigue lives, it is useful to consider a transistion life, which is defined as the number of reversals to failure $(2N_f)_t$ at which the elastic and plastic strain amplitudes are equal.

→ Ie, the transition fatigue life $2N_t$, represents the life at which the elastic & plastic curves intersect [this is the life at which the stabilized hysteresis loop has equal elastic & plastic strain component.

→ By equating elastic & plastic terms, the following expression is derived for the transition life.

$$\frac{\Delta\epsilon_e}{2} = \frac{\Delta\epsilon_p}{2}$$

$$\frac{\sigma_f'}{E}(2N_f)^b = \epsilon_f'(2N_f)^c \quad \text{at } (N_f)_t = N_t \quad N_f = (N_f)_t$$

$$(2N_f)_t = \left(\frac{\epsilon_f' E}{\sigma_f'}\right)^{1/(b-c)}$$

→ At short fatigue lives; i.e when $2N_f << (2N_f)_t$, the plastic strain amplitude is more dominant than the elastic strain amplitude and the fatigue life of the material is controlled by ductility.

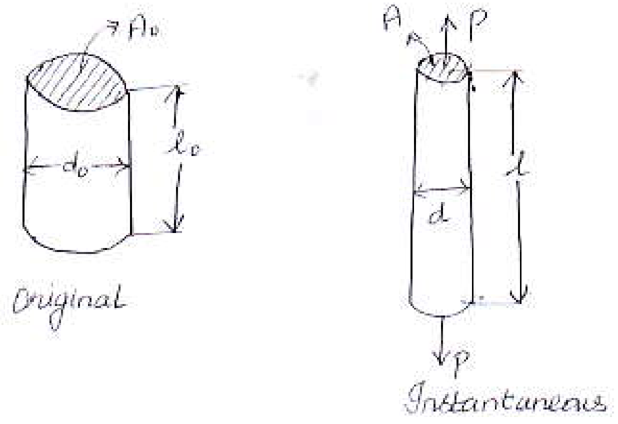
→ At long fatigue lives i.e when $2N_f >> (2N_f)_t$, the elastic strain amplitude is more significant than the plastic strain amplitude & the fatigue life is dictated by the rupture strength.

Monotonic Stress - Strain Behaviour :

A monotonic tension test of a smooth specimen is usually used to determine the Engineering stress - strain behavior of a material where

S = Engineering stress = P/A_0

e = Engineering strain = $\frac{l - l_0}{l_0} = \frac{\Delta l}{l_0}$



P - applied load
l_0 - Original length
d_0 - Original dia
A_0 - Original area
l = Instantaneous diameter
A = Instantaneous area.
d - Instantaneous dia

→ In tension, the true stress is larger than the Engineering stress due to changes in c/s area during deformation

$$\sigma = \text{True stress} = \frac{P}{A}$$

→ Similarly until necking occurs in the specimen, true strain is smaller than Engineering strain.

→ True or natural strain, based on the instantaneous gage length l, is defined as

$$\epsilon = \text{True strain} = \int_{l_0}^{l} \frac{dl}{l} = \ln \frac{l}{l_0}$$

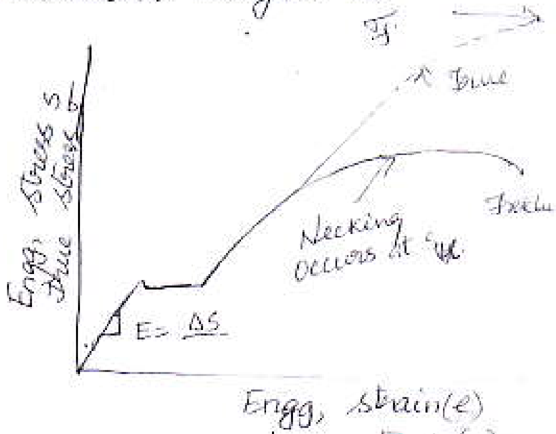
True stress and strain can be related to Eng stress and strain. The instantaneous length is

$$l = l_0 + \Delta l$$

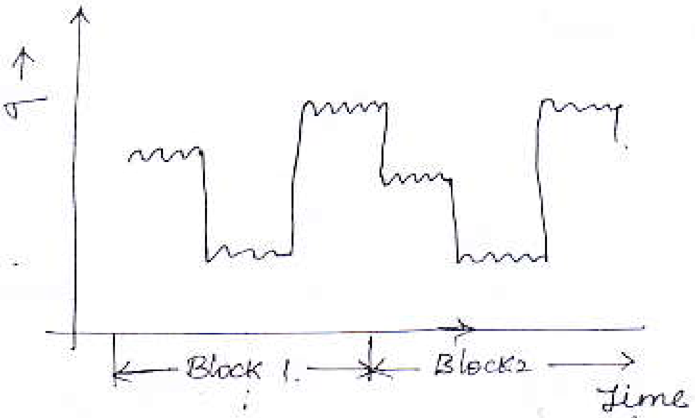
$$\epsilon = \ln \frac{l_0 + \Delta l}{l_0}$$

$$\epsilon = \ln\left(1 + \frac{\Delta l}{l_0}\right)$$

$$\epsilon = \ln(1 + e)$$



Multilevel load history:



Let N_a is the no. of cycles for failure where a particular load S_a is applied.

$$\therefore N_1 > N_2 > N_3 > \dots N_{n-1} > N_n$$

$$\therefore S_1 < S_2 < S_3 < \dots < S_{n-1} < S_n$$

$$\therefore n_q = n_B \cdot \bar{n}_q$$

where $n_B \rightarrow$ no. of blocks in the total period.

$n \rightarrow$ no. of stress levels in a particular block.

$\bar{n}_q \rightarrow$ no. of cycles at stresses S_q in I block.

$n_q \rightarrow$ no. of cycles at stress S_q for total no. of blocks

$$n_q = n_B \cdot \bar{n}_q \rightarrow (1)$$

$$n_{qF} = n_{BF} \cdot \bar{n}_q \rightarrow (2)$$

$n_{BF} \rightarrow$ Total no. of blocks to failures.

$$N_F = n_{1F} + n_{2F} + \dots n_{qF} + \dots n_{hF}$$

$$= (n_{BF} \cdot \bar{n}_1) + (n_{BF} \cdot \bar{n}_2) + (n_{BF} \cdot \bar{n}_3) + \dots +$$

$$(n_{qF} \cdot \bar{n}_q) + \dots (n_{hF} \cdot \bar{n}_h)$$

$$\therefore N_F = \sum_{q=1}^{N} n_{qF} \rightarrow (3a)$$

$\alpha_q \rightarrow$ percentage of no. of cycles at a particular Stress level.

$$\alpha_q = \frac{n_{q_F}}{N_F} \qquad \therefore n_{q_F} = \alpha_q \cdot N_F \rightarrow (3b)$$

Cycle ratio : $\dfrac{n}{N} = \dfrac{\text{no. of cycles applied}}{\text{no. of cycles to failure.}}$

ie Linear Damage $D = \dfrac{n}{N}$ (Say $\dfrac{n}{N} = 0.25$)

In General Linear damage is expressed in terms of %.

$\therefore$ % of Damage = 25%.

where

Applying to Miners theory

$\bar{D}_q = \dfrac{\bar{n}_q}{N_q} \rightarrow$ Linear damage % due to a particular stress level in a particular block.

$$\therefore \bar{D}_B = \sum_{q=1}^{h} \frac{\bar{n}_q}{N_q}$$

For a single block $\bar{D}_B = \sum_{q=1} \bar{D}_q$

$\therefore$ Total damage for n_B block, $D = n_B \cdot \bar{D}_B$

$$\therefore D = n_B \cdot \sum_{q=1}^{h} \frac{\bar{n}_q}{N_q} \quad (\text{from eqn } ①)$$

from ① $\Rightarrow D = \cancel{n_B} \cdot \sum_{q=1}^{h} \dfrac{n_q}{\cancel{n_B}} \cdot \dfrac{1}{N_q}$

$$D = \sum_{q=1}^{h} \cdot \frac{n_q}{N_q} \quad [\text{Total damage for failure}]$$

$D=1$.

If $D=1$ $\quad D = 1 = \sum_{q=1}^{h} \dfrac{n_{q_F}}{N_q}$

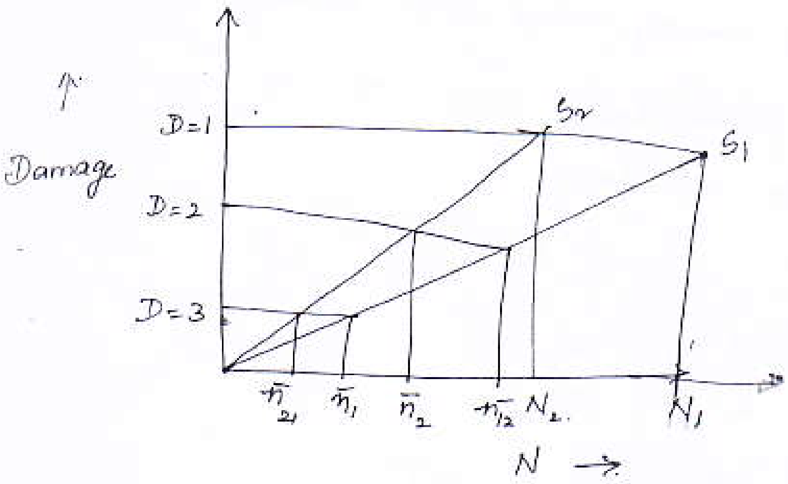
from eqn (3b)

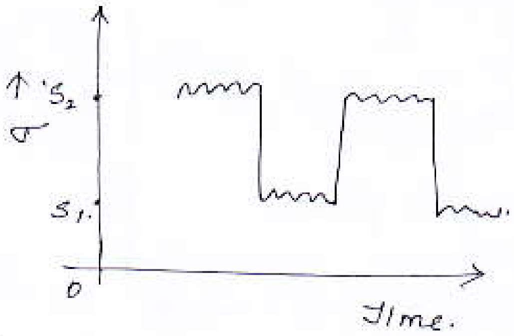
$$n_{q_F} = \alpha_q \cdot N_F$$

$$\therefore D = 1 = \sum_{q=1}^{h} \alpha_q \cdot \frac{N_F}{N_q} = N_F \sum_{q=1}^{h} \frac{\alpha_q}{N_q}$$

$$\therefore \quad N_F = \frac{1}{\sum_{q=1}^{n} \frac{\alpha_q}{N_q}}$$

<u>Structure of Miner's theory:</u>



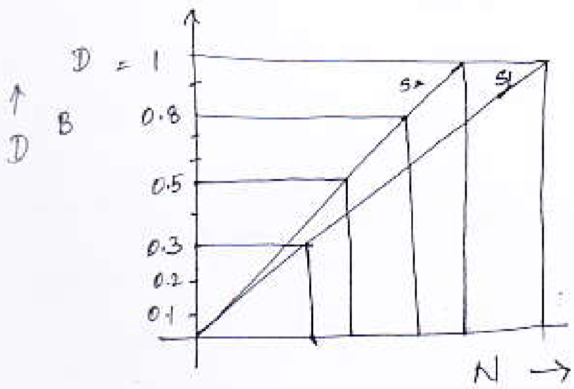


$\bar{n}_{21} \Rightarrow$ 2 – stress level

1 – Damage as in stress level 1

$$D_1 = \left(\frac{\bar{n}_1}{N_1} = \frac{\bar{n}_{21}}{N_2}\right) \Rightarrow \boxed{\bar{n}_{21} = \frac{N_2}{N_1} \cdot \bar{n}_1}$$

$$D_2 = \left(\frac{\bar{n}_2}{N_2} = \frac{\bar{n}_{12}}{N_1}\right) \Rightarrow \boxed{\bar{n}_{12} = \frac{N_1}{N_2} \cdot \bar{n}_2}$$



S_1 & S_2 → 2 different stress levels.

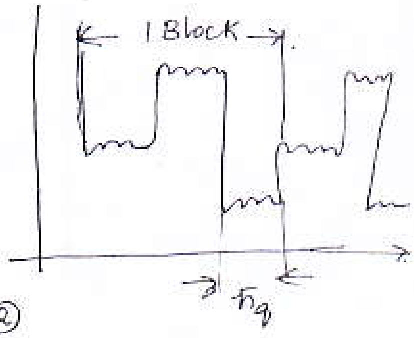
For producing 0.8 or 80% on S_2 level, we can apply either 50% damage on S_2 & then 30% damage on S_2 or 50% damage on S_2 and 30% damage on S_1 level.

$N_1, N_2 \rightarrow$ individual max. no. of cycles reqd., to produce failure.

$$\bar{n}_{2B} = \bar{n}_2 + \bar{n}_{21} = n_2 + \frac{N_2}{N_1}\bar{n}_1$$

$$\bar{n}_{2B} = N_2\left[\frac{\bar{n}_1}{N_1} + \frac{\bar{n}_2}{N_2}\right]$$

$$\frac{\bar{n}_{2B}}{N_2} = \frac{\bar{n}_1}{N_1} + \frac{\bar{n}_2}{N_2} \rightarrow (1)$$



$$N_2 = n_{BF} \cdot \bar{n}_{2B}$$

n_{BF} ↓ no. of blocks to failure.

$$n_{BF}\left(\frac{\bar{n}_1}{N_1} + \frac{\bar{n}_2}{N_2}\right) = 1 \rightarrow (2)$$

$$n_{qF} = n_{BF} \cdot \bar{n}_q$$

$$n_{1F} = n_{BF} \cdot \bar{n}_1$$
$$n_{2F} = n_{BF} \cdot \bar{n}_2$$

$$(2) \Rightarrow \boxed{1 = \frac{n_{1F}}{N_1} + \frac{n_{2F}}{N_2}}$$ → This is for two stress level.

∴ For h stress levels

$$\sum_{q=1}^{h} \frac{n_{qF}}{N_q} = 1$$

WKT

$$\frac{N_q}{N_h} = \left(\frac{S_h}{S_q}\right)^b \Rightarrow N_q = N_h\left(\frac{S_h}{S_q}\right)^b$$ → b depends on material.

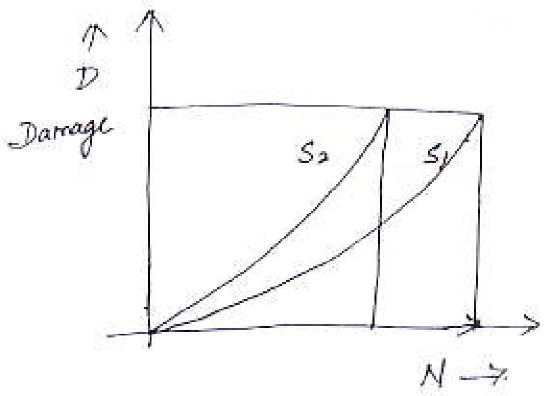
$$\sum_{q=1}^{h} \frac{n_{qF}}{N_h}\left(\frac{S_q}{S_h}\right)^b = 1$$

WKT

$$N_F = \frac{1}{\sum_{q=1}^{n} \frac{\alpha_q}{N_q}}$$

$$N_F = \frac{1}{\sum_{q=1}^{n} \frac{\alpha_q}{N_n} \left(\frac{S_q}{S_n}\right)^{b}}.$$

Modified Miner's theory:



As N ↑ses, damage rate ↑ses ∴

S_1 & S_2 are curved. $\frac{dD}{dn}$ depends on the no. of cycles applied & hence slope ↑ses rises with ↑se in no. of cycles.

$$D = \left(\frac{n}{N}\right)^{x} \qquad \frac{dD}{dn} = \frac{x}{N^{x}} \cdot n^{x-1}$$

$$D_1 = \left(\frac{n}{N_1}\right)^{x} \qquad \frac{dD}{dn} = \frac{x}{N} \cdot \left(\frac{n}{N}\right)^{x-1}$$

$$D_2 = \left(\frac{n}{N_2}\right)^{x}. \qquad \frac{d^2D}{dn^2} \neq .$$

$$D_1 = \left(\frac{\bar{n}_1}{N_1}\right)^{x} = \left(\frac{\bar{n}_{21}}{N_2}\right)^{x}$$

$$\bar{n}_{21} = \frac{N_2}{N_1} \cdot \bar{n}_1 \Rightarrow$$ So in the modified Miner's theory, the result is not changed by varying the initial term.

So even by changing the linearity of the curves the Miner's theory is not affected.

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Groover's theory

Here the no. of cycles to failure is splitted into two parts

$$N_f = N_f' + N_f''$$ → (no. of cycles reqd to propagate the crack to a critical level).

N_f' ↓ No. of cycles reqd to initiate a crack (or) crack nucleation

WKT

$$N_f = \sum_{q=1}^{h} n_{qF} \quad \text{where } n_{qF} = n_{eF} \cdot \bar{n}_q$$

$$N_f' = \sum_{q=1}^{h} n_{qF}'$$

$$N_f'' = \sum_{q=1}^{h} n_{qF}''$$

also WKT, $n_{qF} = \alpha_q \cdot N_F$ (% level).

$$\therefore n_{qf}' = \alpha_q \cdot N_F' \rightarrow (1)$$

$$n_{qF}'' = \alpha_q \cdot N_F'' \rightarrow (1a)$$

No. of cycles reqd., at stress level S_q to produce failure.

$$N_q = N_q' + N_q''$$

$$\frac{N_q'}{N_q} + \frac{N_q''}{N_q} = 1$$

Let $\frac{N_q'}{N_q} = a_q \rightarrow (2)$

$$a_q + \frac{N_q''}{N_q} = 1$$

$$\boxed{\therefore 1 - a_q = \frac{N_q''}{N_q}}$$

By Miner's theory $\sum_{q=1}^{h} \frac{n_{qf}'}{N_q'} = 1 \rightarrow (3)$

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Sub ① & ② in eqn ③

$$\sum_{q=1}^{h} \frac{N''_{qf}}{N''_{q}} = 1 \rightarrow (3a)$$

Sub ① & ② in eqn ③

$$\sum_{q=1}^{h} \frac{\alpha_q \cdot N_F'}{N_q'} = 1$$

$$\sum_{q=1}^{h} \frac{\alpha_q \cdot N_F}{\left(\frac{N_q'}{N_q}\right) N_q} = 1$$

$$\sum_{q=1}^{h} \frac{\alpha_q \cdot N_F'}{a_q \cdot N_q} = 1$$

$$N_F = \frac{1}{\sum_{q=1}^{h} \frac{\alpha_q}{a_q \cdot N_q}} \rightarrow (4)$$

using (1a), ② & (3a)

$$N_F' = \frac{1}{\sum_{q=1}^{h} \frac{\alpha_q}{(1-a_q) \cdot N_q}}.$$

2

UNIT-II

1. Estimate the expected life of the compressor if it is operated according to the following schedule.

(i) 10 min - @ 30 rpm @ 25×10^3 N/mm² for 400 cycles time with fatigue life of 50×10^3 hrs.

(ii) 20 min @ 25 rpm @ 15×10^3 N/mm² for 300 cycles time with fatigue life of 5000 hrs.

(iii) 30 min @ 10 rpm @ 1×10^4 N/mm² for 500 cycles time with fatigue life of 2000 hrs.

Data are obtd from S-N curve. N - total life of stress cycles.

N_1, N_2, N_3 - Individual life of σ_1, σ_2 & σ_3

n_1, n_2, n_3 - No of times of cycle.

Given

$\sigma_1 = 25 \times 10^3$ N/mm² ; $t_1 = 10$ min ; $n_1 = 400$; $N_1 = 50 \times 10^3$ hrs

$\sigma_2 = 15 \times 10^3$ N/mm² ; $t_2 = 20$ min ; $n_2 = 300$; $N_2 = 5000$ hrs

$\sigma_3 = 1 \times 10^4$ N/mm² ; $t_3 = 30$ min ; $n_3 = 500$; $N_3 = 2000$ hrs.

Total working period is 1 hr. is

Total no of cycle for 1 hr is

$\frac{n_1}{N_1} \Rightarrow$ fatigue damage for particular cycle

$$n = n_1 + n_2 + n_3$$

$$= 400 + 300 + 500 = 1200 \text{ cycles/hr.}$$

The proportional damage for each cycle is :

$$\alpha_1 = \frac{n_1}{n} =$$

$$\alpha_2 = \frac{n_2}{n} =$$

$$\alpha_3 = \frac{n_3}{n} =$$

Total life is

$$\frac{\alpha_1}{N_1} + \frac{\alpha_2}{N_2} + \frac{\alpha_3}{N_3} = \frac{1}{N}.$$

Life in hrs is

$$N = \frac{3773.56 \text{ cycles}}{1200 \text{ cycles/hr}}$$

$$= 3.1446 \text{ hrs.}$$

2) A steel plate subjected to completely reversed stress for the period of 50 sec. The stress values are

4 cycles @ 500 N/mm²
3 " 750 "
2 " 250 "

9 cycles acting for the period of 50 sec, find the fatigue life for the plate for the corresponding individual life $10^3, 10^5, 10^2$ for respective stress cycle.

$\sigma_1 = 500\ N/mm^2$; $n_1 = 4$
$\sigma_2 = 750$ " ; $n_2 = 3$
$\sigma_3 = 250$ " ; $n_3 = 2$

$N_1 = 10^3$; $N_2 = 10^5$; $N_3 = 10^2$ $t = 50$ sec.

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} \leq 1 \rightarrow \text{for safety}$$

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = 0.02403 < 1$$

So the compt will not fail.

For calculating the life of the plate.

The value 0.02403 is for 50 sec time period.

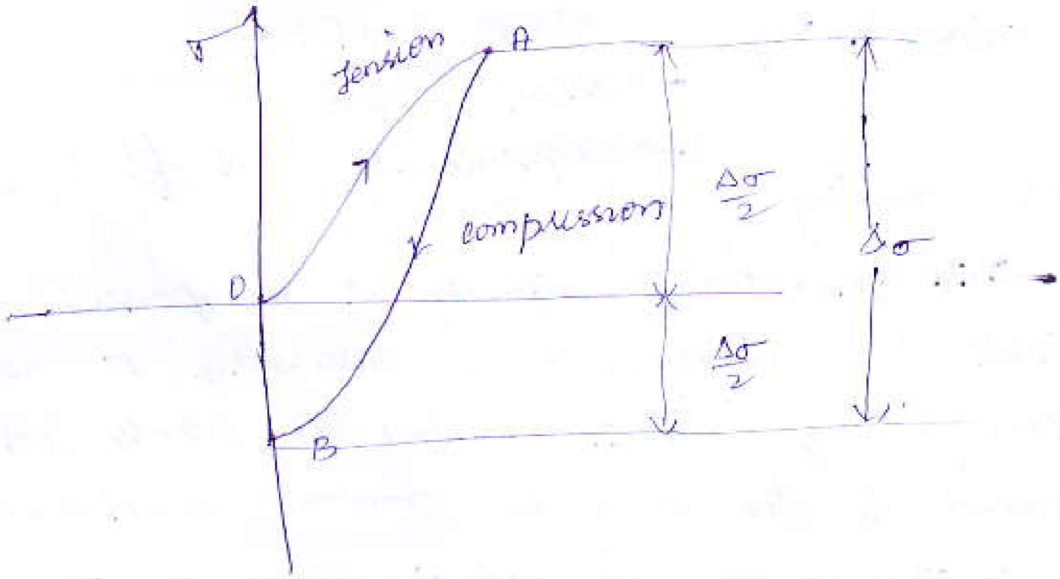
We can design upto the value 1.

So designing for the Miner's eqn for the value of 1

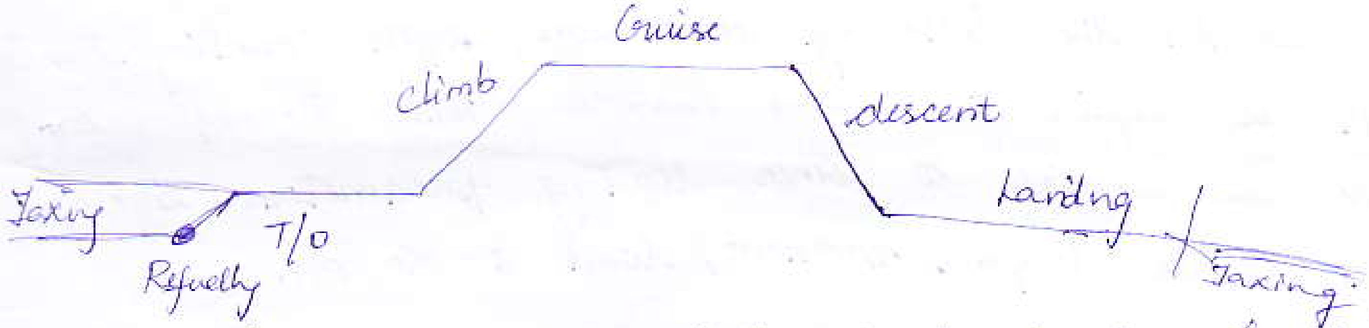
$$\text{Fatigue life} = \frac{50\ \text{sec}}{0.02} = \frac{50/3600}{0.02}$$

$$= 0.69\ \text{hrs.}$$

Bauschinger effect:



Analysis of load hysteresis:



When component subjected to random loading we need to find the load values at each time period. For eg in A/c structures, it is subjected to different random loads at various periods. ie Taxing, T/O, climb, cruise, descent, landing.

The fatigue loads are recorded by either stress or strain w.r.t time period. By using cyclic loading algorithm, the datas are extracted from the recorded values. By using cyclic counting algorithm, the load levels at each time period is calculated. ~~while~~ ca

While calculating, the three important factors are considered, viz → Stress amplitude
→ Mean Stress
→ Sequence of load fluctuation

Cycle counting:

→ To predict the life of a component subjected to a variable load history, it is necessary to reduce the complex history into a number of events which can be compared to the available constant amplitude test data.

→ This process of reducing a complex load history into a number of constant amplitude events involves what is termed as cyclic counting.

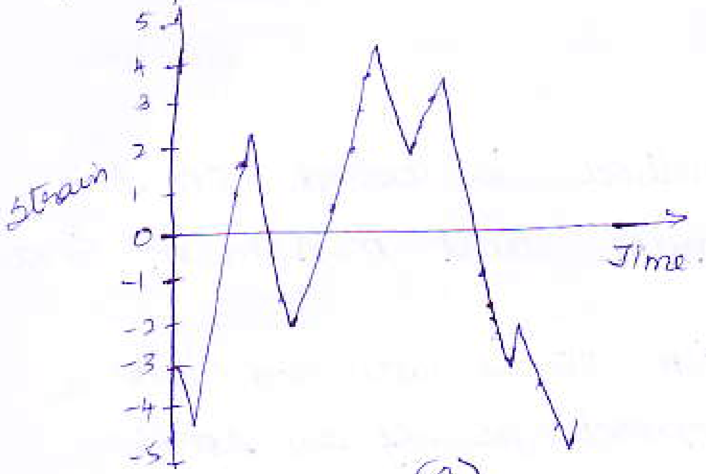
→ In the following discussion, cycle counting techniques will be applied to strain histories, these techniques can also be applied to other loading parameters, such as stress, torque, moment, load & so on.

Various cyclic counting algorithms

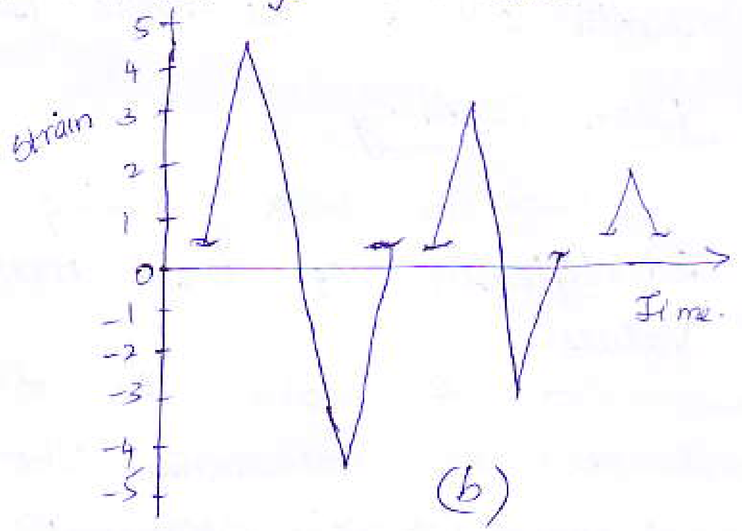
* Level - crossing counting method
* peak counting method
* Simple - range counting method
* Rain-flow counting method.

Level-Crossing counting

→ In this procedure the strain axis of the strain-time plot is divided into a number of increments



(a)



(b)

→ A count is then recorded each time a positively sloped portion of the strain history crosses an increment located above the reference strain

→ Similarly, each time a negatively sloped portion of the strain history crosses an increment located below the reference strain, a count is made.

→ In addition, crossings at the reference strain by a positively sloped portion of the strain history are also counted.

→ Fig (a) shows a sample strain history & the resulting level-crossing counts., Here strain has been used as the reference strain value.

→ Once the counts are determined, they must be combined to form completed cycles.

→ A variety of methods are available for the combining of counts to obtain completed cycles.

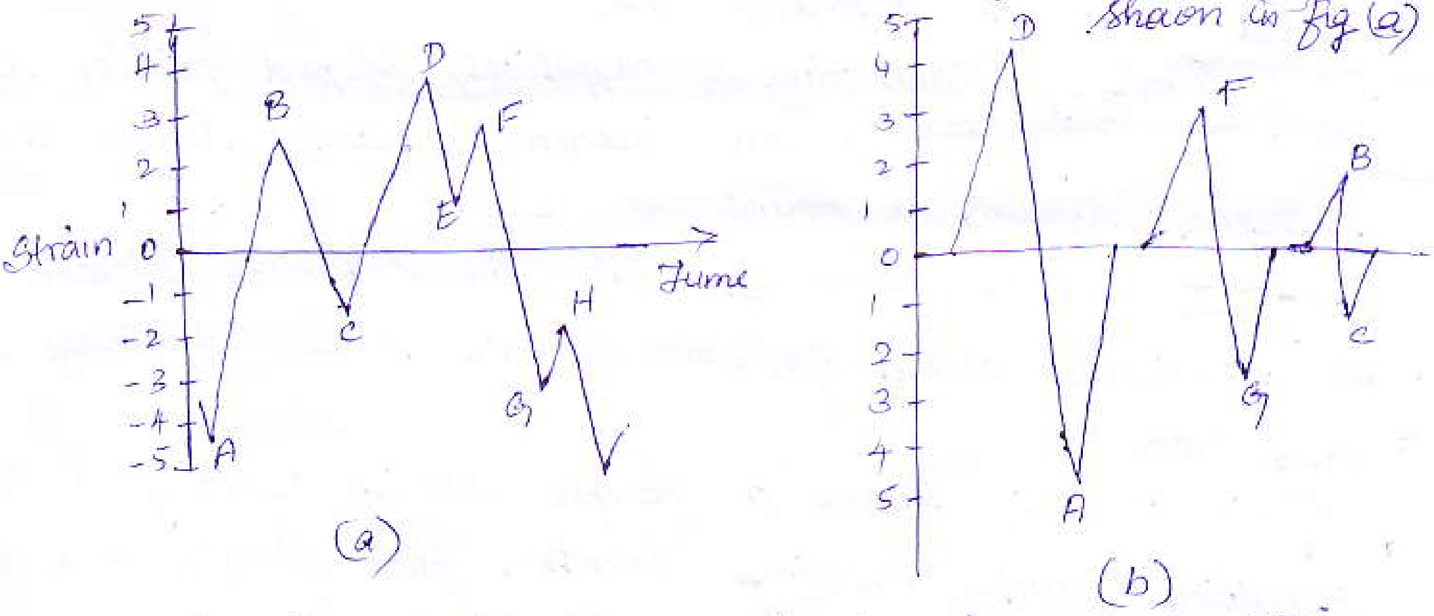
→ The most damaging combinations of counts, is obtd by first forming the largest possible cycle.

→ The next largest cycle possible is then formed by using the remaining counts available and so on untill all counts have been used.

Peak counting:

→ The peak counting method is based on the identification of local maximum and minimum strain values.

→ To begin, the strain axis is divided into a number of increments. The positions of all local maximum (peak) strain values above the reference strain are tabulated & the positions of all local minimum strain values below the reference strain. Shown in fig (a)



→ Here also zero strain has once again been used as the reference strain value.

→ As with the level-crossing method, once counts have been obtained, they must be combined to form complete cycles in order to perform a fatigue analysis.

→ The most damaging history in terms of fatigue is obtd by first combining the largest peak with the largest valley.

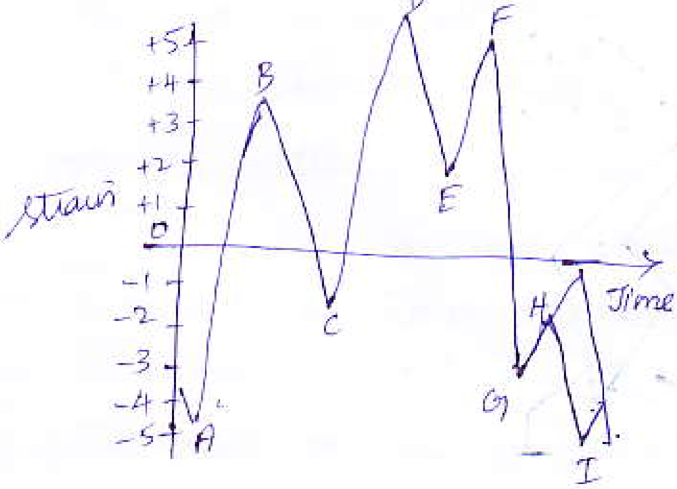
→ The Second largest cycle is then formed by combining the largest peak and valley of the remaining counts. This process is continued untill all counts have been used.

→ Fig 'b' shows the resulting completed cycles obt'd from using this procedure for the peak count obt'd for the strain history shown in fig 'a';

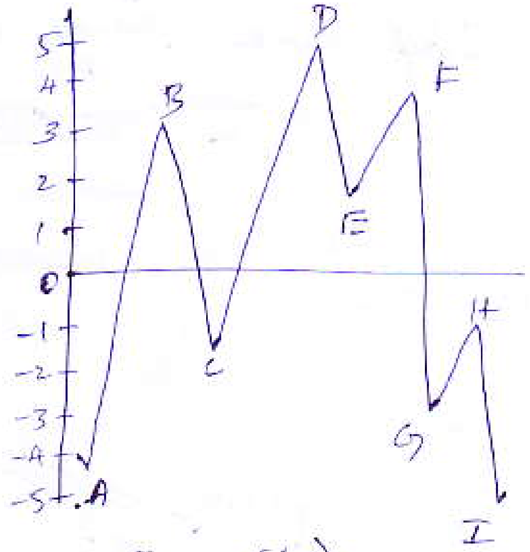
SIMPLE RANGE COUNTING :

→ With this method the strain ~~of~~ range between successive reversals is recorded. In determining counts, if both positive ranges (valleys followed by peaks) and negative ranges (peaks followed by valleys) are included, each range is considered to form one-half cycle.

→ If just positive or negative ranges are recorded, each is considered to form one full cycle. A cycle count completed using this method for a sample strain history is shown in fig (a) below.



(a)



(b)

→ Here both positive and negative ranges were counted.

→ In fig (b), each range is considered to form one-half cycle.

→ In using the simple-range counting method, the mean value of each range is often recorded

→ This information is then combined with the determined range values in the form of a 2D matrix.

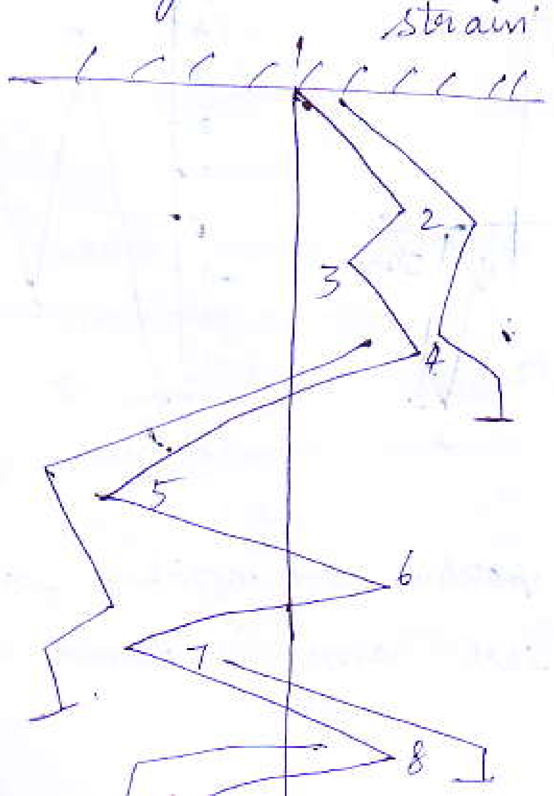
→ In performing a fatigue analysis, mean stress effects are then included. When mean values are recorded, this cycle counting procedure is then called the simple-range-mean counting method.

RAIN FLOW COUNTING METHOD:

→ It seperates high & low amplitude cycles and record the physically meaningful way by reducing the complex loading data into serious of threshold nominal stress.

→ This uses the concept of rain flowing down through every roof starting at highest point and dripping off at each extremities as shown in fig

→ The length of each loading is terminated as half cycle range



BAUSCHINGER EFFECT:

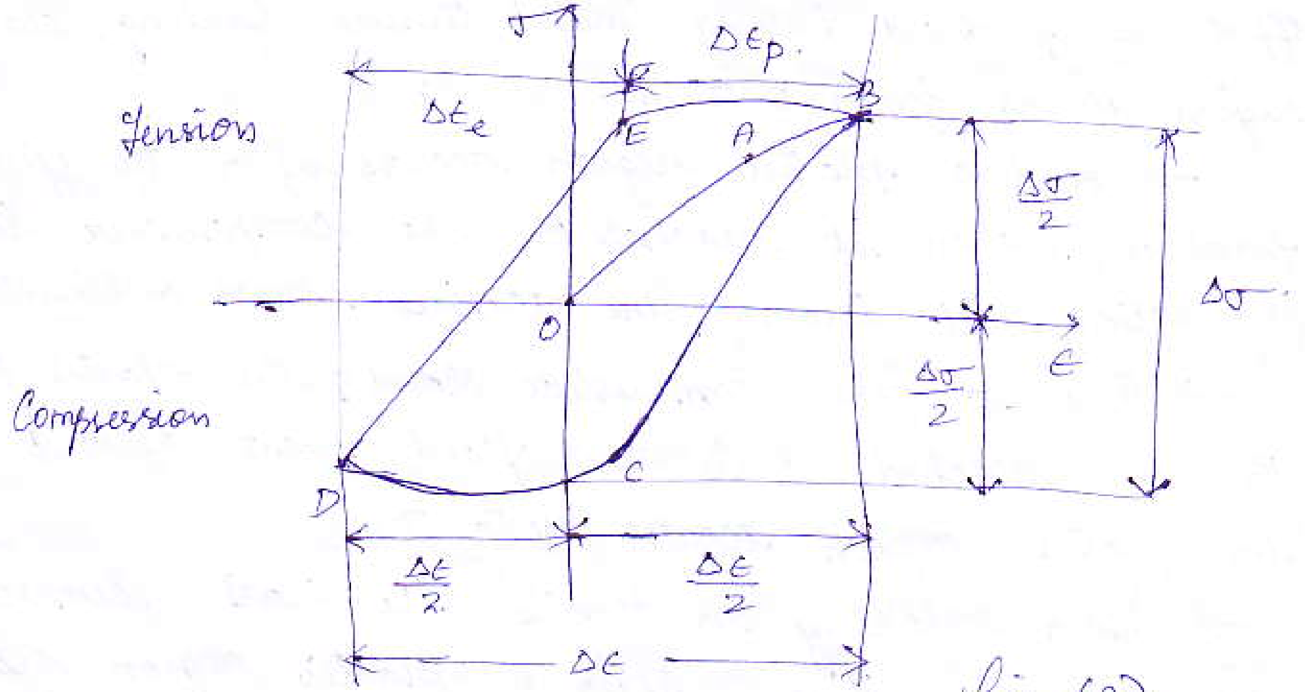


fig (A)

$$\Delta\epsilon = \Delta\epsilon_e + \Delta\epsilon_p$$

$$\Delta\epsilon = \frac{\Delta\sigma}{E} + \Delta\epsilon_p$$

→ When cyclic stress & cyclic strain is acting, Bauschinger effect is produced on the material.

→ Due to cyclic strain control fatigue, a component expands and contracts, response to the fluctuation load.

→ It is normally occurs in thermal cyclic and reverse bending between the fixed displacement.

→ The localised plastic strain are acting at a notch subjected to either cyclic stress or cyclic strain condition.

→ It will produce strain controlled conditions near the root of the notch due to constrained effect of larger surrounding mass. & exactly elastic deformation occurs in the material.

→ Due to constant strain cycle, the Bauschinger effect is produced. During initial Tensile loading, elastic region occurs from O to A.

→ And a plastic region occurs after the yield point A, when it reaches B, the compressive load is acting, it lowers the tensile stress & tensile strain

→ Due to the compressive load, the elastic region BC is reached. C is the yield point, after which the plastic region occurs upto D.

→ Again reloading in tensile, the elastic curve reaches a point E. After E, plastic region starts upto B or more than B, depends upon the nature of the material.

→ The dimensions of the hysteresis loop are determined by its width ($\Delta\varepsilon$) & its height ($\Delta\sigma$).

→ $\Delta\epsilon$ represents total strain range & $\Delta\sigma$ represents Total stress range. The total strain range $\Delta\epsilon$ consists of $\Delta\epsilon_e$ and $\Delta\epsilon_p$ where

$\Delta\epsilon_e$ - strain in elastic ; $\Delta\epsilon_p$ - strain in plastic.

→ The plastic region $\Delta\epsilon_p$ is very difficult to find out. The plastic deformation is not completely reversible.

→ The modification of the structure occurs during cyclic straining process. Depending on the initial state of material, That may undergo cyclic hardening, cyclic softening or remain cyclically stable.

Cyclic strain hardening & softening:

Refer to the book, "Fundamentals of metal fatigue Analysis" by Julie. A. Bannantine, Jess. J. Comer & James L. Handrock pg no. 48 to 52.

Problems:

1. A steel rod subjected to torsional load of 5 cycles at $1000 N/mm^2$, 3 cycles at $500 N/mm^2$, 3 cycles at $250 N/mm^2$. The load is acting for the period of 30 min. Corresponding life cycle for above stresses are 3×10^5, 1×10^3, 1.2×10^2 respectively. The endurance limit of the steel is $400 N/mm^2$. Evaluate the fatigue life of the component.

GIVEN:

$n_1 = 5$; $N_1 = 3\times10^5$; $t = 30$ min.

$n_2 = 3$; $N_2 = 10^3$

$n_3 = 3$; $N_3 = 1.2\times10^2$,

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = 0.028$$

if $\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} < 1$, the component will not fail.

Now

$$\text{fatigue life} = \frac{30 \text{ minutes}}{0.028} = \frac{\frac{30}{60}}{0.028} \text{ in hrs}$$

$$= 17.86 \text{ hrs.}$$

CUMULATIVE DAMAGE:

→ The principle of stress-based characterization of total fatigue life are only relevant for constant amplitude fatigue loading.

→ In reality, however engineering components are invariably subjected to varying cyclic stress amplitudes, mean stress and loading frequencies.

→ A simple criteria for predicting the extend of fatigue damage induced by a particular block of constant amplitude cyclic stresses, in a loading sequence consisting of various blocks of different stress amplitudes is provided by the so-called so-called palmgren-Miner cumulative damage rule.

→ Assumptions of linear damage rule are

(i) The no. of stress cycles imposed on a component, expressed as a percentage of the total number of stress cycles of the same amplitude necessary to cause failure, gives the fraction of damage.

(ii) The order in which the stress blocks of different amplitudes are imposed does not affect the fatigue life.

(iii) Failure occurs when the linear sum of the damage from each load level reaches a critical value.

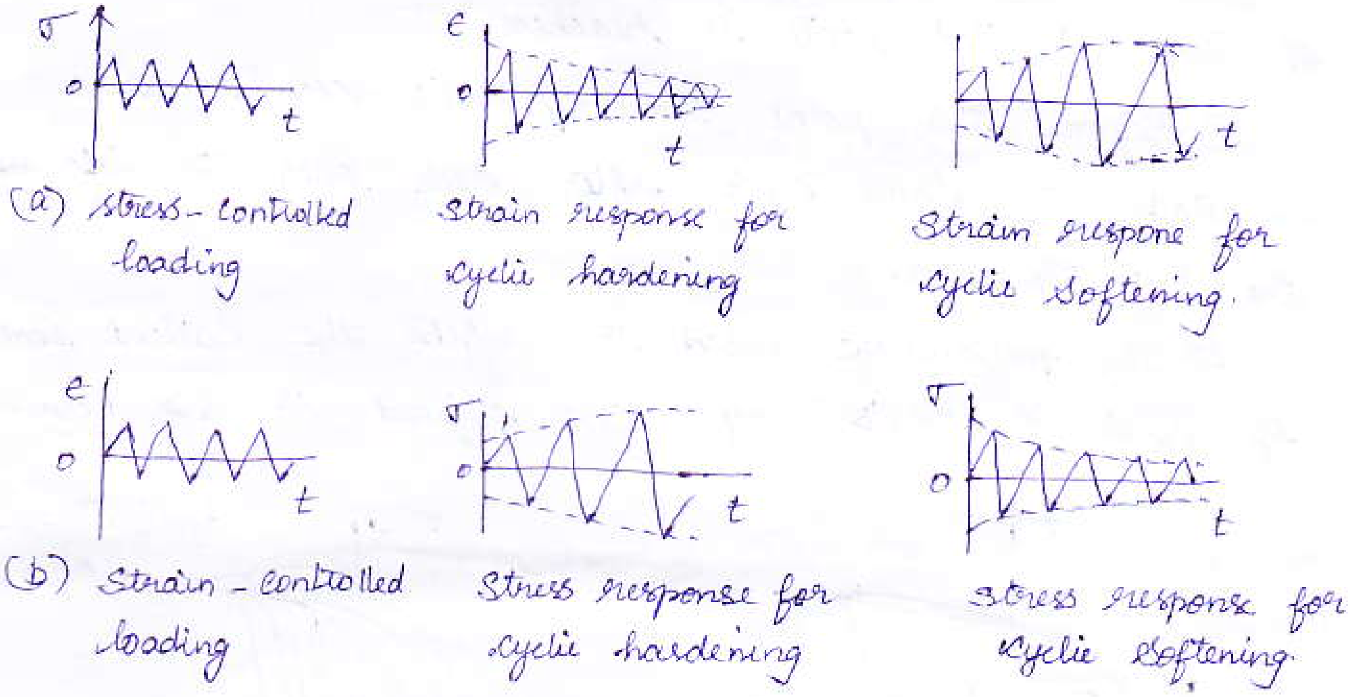
→ If n_i is the no. of cycles corresponding to the i^{th} block of constant stress amplitude σ_{ai} in a sequence of 'm' blocks and if N_{fi} is the no. of cycles to failure at σ_{ai}, then the palmgren-Miner damage rule states that failure would occur when

$$\sum_{i=1}^{m} \frac{n_i}{N_{fi}} = 1$$

CYCLIC HARDENING AND SOFTENING: ~~A~~ ;

→ The uniaxial deformation of Engg. alloys subjected to cyclic loads is usually characterized by the cyclic stress - strain curve (CSS curve)

→ The transient phenomena typically associated with cyclic deformation are schematically shown in fig below.



(a) stress - controlled loading; strain response for cyclic hardening; Strain response for cyclic softening.

(b) Strain - controlled loading; Stress response for cyclic hardening; stress response for cyclic softening.

~~Transient associated with~~

fig — Transient effects in fatigue.

→ In the case of constant amplitude, fully reversed stress control in fig (a), cyclic hardening or softening of the material is reflected by a reduction or an increase respectively, in the axial strain amplitude.

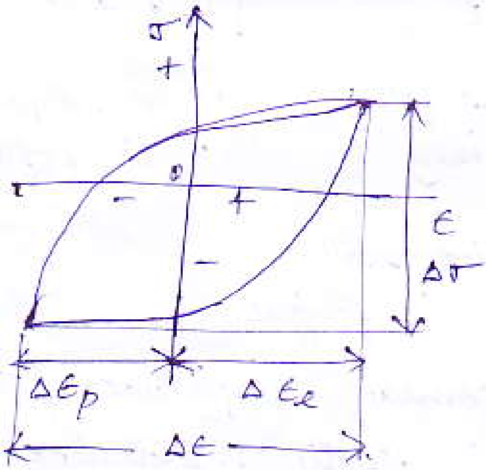
→ Similarly, under constant amplitude, strain-controlled fatigue loading, cyclic hardening or softening of the material causes an increase or decrease respectively in the axial stress amplitude shown in fig (b).

→ In both stress-controlled and strain-controlled fatigue, the respective strain amplitude and stress amplitude reach a stable saturation value after an initial 'shakedown' period.

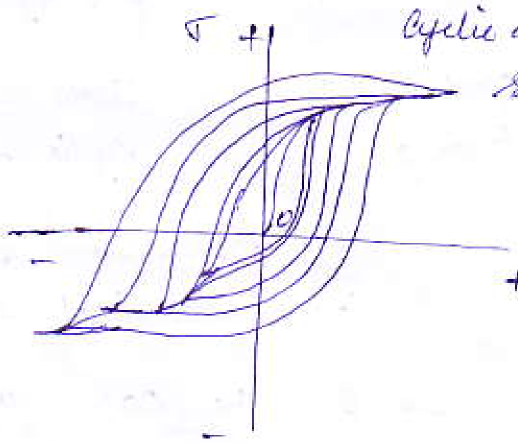
→ This saturation state gives rise to stable hysteresis loops. During shake-down, there is a continual change in dislocation substructure until a stable configuration of the saturated state is reached.

→ Beyond this point, the hysteresis loop remains essentially the same cycle after cycle over the life of the test specimen.

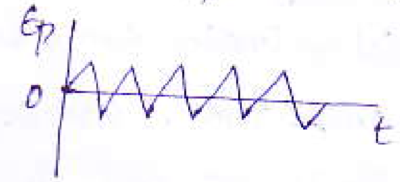
→ The parameters used to describe the salient features of cyclic hysteresis loops are defined in fig below (a)



(a) stable hysteresis loop

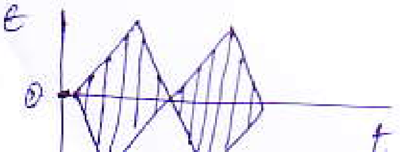


(b)



constant plastic strain limit

multiple step increasing plastic strain limit after saturation at each step.



incremental step, repeated patterns consisting of linearly increasing

→ The locus of the tips of stable hysteresis loops provides the cyclic stress-strain curve shown in fig(b).

→ stress and strain-controlled fatigue represent extremes of fully uncontrained and fully contrained loading conditions.

→ In real Engg., components, there is usually some structural constraint of the material at fatigue-critical sites It thus seems appropriate to characterize fatigue response of Engg. materials on the basis of data obtained under strain-controlled fatigue rather than cyclic stress-controlled conditions.

→ Strain-controlled tests have gained increased use in the determination of CSS curves for Engg. alloys. Three commonly used strain-controlled test methods are indicated in fig (c).

→ In constant amplitude test, the specimen is cycled within a constant plastic strain limit (until failure) to obtain a single stable hysteresis loop.

→ Multiple test specimens are needed to determine the entire CSS curve using this method. In the multiple step method, a specimen is cycled b/w constant plastic strain limits until a saturation loop results.

→ Then the plastic strain limits are incremented until another stable hysteresis loop is obtd. This process is ~~incremented until another stable hysteresis loop is obtd~~ continued untill the entire CSS curve is measured from a single ~~step~~ test specimen.

→ In the incremental step method, the specimen is subjected repeatedly to a strain pattern comprising linearly +ing & -ing amplitudes, from zero to a certain max total strain.

→ The monotonic stress - strain behaviour of ductile solids under uniaxial tension is generally represented by a constitutive law, such as Ramberg - osgood relationship

$$\epsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{A}\right)^{1/n_m}$$, where E - Young's modulus

A is a constant commonly referred to as the monotonic strength coefficient, ϵ is the uniaxial strain, σ is the uniaxial stress, n_m is the strain hardening exponent.

→ The typical range of n_m for alloys is 0 - 0.5.

In an analogous fashion, the cyclic stress - strain response is characterized by the relationship

$$\frac{\Delta \epsilon}{2} = \frac{\Delta \sigma}{2E} + \left(\frac{\Delta \sigma}{2A'}\right)^{1/n_f}$$

where A' is the cyclic strength coefficient, & n_f is the cyclic strain hardening exponent.

→ For most metal n_f varies b/w 0.1 & 0.2 despite Va differences in their cyclic hardening and softening characteristic.

→ As a general rule of thumb, well-annealed, polycrystalline metals of high purity exhibit cyclic hardening due to dislocation multiplication, as evidenced by an increase in the stress amplitude with fatigue cycles, Work - hardened materials undergo strain softening under strain ~~softening~~ cyclic loading.

→ The rearrangement of pre strain - induced dislocation networks due to fatigue causes cyclic softening.